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THE DETERMINATION OF ABSOLUTE DIRECTIONS IN SPACE WITH ARTIFICIAL SATELLITES

by

George Veis

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WITH ARTIFICIAL SATELLITES[1]

by

George Veis[2]

10440

Abstract.--The Smithsonian Astrophysical Observatory has modified the Baker-Nunn cameras to perform simultaneous observations. When only two stations are involved in simultaneous observing, the directions in an absolute system of reference of the line connecting the two stations can be determined. Fifty-six pairs of simultaneous observations between stations Villa Dolores, Argentina; Arequipa, Peru; Curaçao, Netherlands Antilles; Jupiter, Florida; and Organ Pass, New Mexico, indicate that an accuracy of better than 1" of arc can be expected.

AUTHOR

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[2]Smithsonian Astrophysical Observatory; and the National Technical University, Athens, Greece.

Introduction

If artificial satellites are used as triangulation points and simultaneous observations are made, a space triangulation scheme is possible and permits a three-dimensional determination of the position of a station.

To perform, however, a space triangulation with only angular measurements, we need just as in the case of a regular two-dimensional triangulation, a baseline to give the scale, as well as the coordinates, of at least one point to be used as the origin. To orient a two-dimensional triangulation we also need the orientation of the baseline (or any other line). In a three-dimensional triangulation in which absolute directions have been observed, the space orientation of the whole net is obtained automatically, but the scale can be determined only if at least one distance is known. Such a distance can be either that between two ground stations, obtained by classical methods of triangulation, or that between one station and the satellite, obtained by direct range measurement. The position for all the unknown stations will, of course, be obtained in the same reference system in which the coordinates of the origin are given (Veis, 1960).

If the reference system is defined by the coordinates of only one point (that of the origin), its orientation by a consistent system of directions (the same as that used for the observations) and the scale is given by one baseline; the reference system is uniquely determined. If there is more than one known station in the space triangulation net, their coordinates must be consistent with the system of directions used for the observations, and with the unit of any possible distance measurements.

If only two known stations are involved in a three-dimensional triangulation, the scale will, of course, be defined uniquely by the distance between the stations as obtained from their coordinates, but the direction of the line connecting the two stations must be consistent with the system of directions used for the observations (Veis, 1963).

The Smithsonian Astrophysical Observatory has modified the Baker-Nunn cameras to perform simultaneous or, more correctly, quasi-simultaneous observations. However, the locations of and distances between the stations of the net do not easily permit simultaneous visibility from three stations of satellites now in orbit.

If only two stations at a time are involved in such a scheme, no solution is possible for the relative position of the one station with respect to the other. In this case, we can easily prove that the solution will be the equation of the line that connects the two stations. Since the simultaneously observed directions to the satellite will refer to an absolute and universal system, the equation of the line will also be in that same system. Therefore, although the position of the unknown station cannot be determined, the direction of the line connecting the known and the unknown stations can be obtained (Kukkamaki 1959).

The above has a rather simple geometrical interpretation, which offers a simple graphical solution for the directions.

<u>The geometric interpretation</u>

In figure 1, let A be the known station, B the unknown station, and S the i-th position of a satellite when a pair of simultaneous observations was made. Furthermore, let p_i q_i be the observed directions from stations A and B, respectively, expressed in direction cosines in a nonrotating terrestrial system. The vectors $\underset{\sim}{p}$ and $\underset{\sim}{q}$ define a plane that can be defined by the normal vector

$$\underset{\sim}{n} = \underset{\sim}{p} \times \underset{\sim}{q} .$$

Every pair of simultaneous observations defines a plane. The intersection of such planes will give the line that connects the stations A and B.

Let $\bar{B}$ be an approximate position for the unknown station B, and let ω be a plane perpendicular to the line $A\bar{B}$ at $\bar{B}$. Every plane, defined by a pair of simultaneous observations, will intersect this plane along a line μ (fig. 2). Two pairs will give two lines μ_1 and μ_2 that will intersect at the correct position of B on the plane ω, i.e., the intersection of the correct directions AB with the plane ω, if no errors in the observations are assumed.

It is obvious that in the case of the two observed pairs, the direction of the line connecting the two stations will be

$$\underset{\sim}{c} = \underset{\sim}{n}_1 \times \underset{\sim}{n}_2$$

where $\underset{\sim}{n}_1$ and $\underset{\sim}{n}_2$ are the normal vectors of the two planes.

If we have more than two pairs of simultaneous observations, and there are unavoidable errors in the observations, all the lines will not intersect at the same point and an adjustment will be needed.

Let $\underset{\sim}{a}$ and $\bar{\underset{\sim}{b}}$ denote the vectors in the terrestrial system that define A and $\bar{B}$, and let $\bar{\underset{\sim}{c}}$ be the vector $\bar{\underset{\sim}{b}}-\underset{\sim}{a}$. If the position $\bar{B}$ is correct, and we assume no errors in the observed directions, then $\underset{\sim}{n} \cdot \bar{\underset{\sim}{c}} = 0$, or $[\underset{\sim}{q}\underset{\sim}{p}\bar{\underset{\sim}{c}}] = 0$, since $\underset{\sim}{p}$, $\underset{\sim}{q}$, and $\bar{\underset{\sim}{c}}$ would be coplanar. However, since $\bar{B}$ is at an erroneous position $\underset{\sim}{p}$, $\underset{\sim}{q}$, $\bar{\underset{\sim}{c}}$ will not be coplanar and $n \cdot \bar{c} \neq 0$; then let $\underset{\sim}{n}^{o} \cdot \underset{\sim}{c}^{o} = \epsilon$, the cosine of the angle between $\underset{\sim}{n}$ and $\bar{\underset{\sim}{c}}$, ($\underset{\sim}{n}^{o}$ and $\bar{\underset{\sim}{c}}^{o}$ being the unit vectors of $\underset{\sim}{n}$ and $\bar{\underset{\sim}{c}}$.) Since this angle is very close to 90^{o}, ϵ will be the small angle between $\bar{\underset{\sim}{c}}$ and the plane defined by $\underset{\sim}{p}$ and $\underset{\sim}{q}$, if we assume this plane to pass from point A. The distance of the plane from $\bar{B}$ will then be $e = \lambda \cdot \sin \epsilon$, where $\lambda = |\bar{\underset{\sim}{c}}|$ is the distance between A and B; or $\underset{\sim}{e} = \bar{\underset{\sim}{c}} \cdot \underset{\sim}{n}^{o}$. This will also be the distance of $\bar{B}$ from the line μ on the plane ω. To obtain the orientation of μ, we introduce a right-hand reference system 1 centered at $\bar{B}$ and oriented so that the 3-axis will be parallel to $\underset{\sim}{c}$ (the 1-2 plane coincides with the ω plane) and the 2-axis will be on the plane of $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

Let $\underset{\sim}{g}_1, \underset{\sim}{g}_2, \underset{\sim}{g}_3$ denote the three axes in the terrestrial system. Hence

$$\underset{\sim}{g}_3 = \bar{\underset{\sim}{c}}^{o} ,$$

$$\underset{\sim}{g}_1 = \frac{1}{|\underset{\sim}{a} \times \underset{\sim}{b}|} \cdot \underset{\sim}{a} \times \underset{\sim}{b} ,$$

$$\underset{\sim}{g}_2 = \underset{\sim}{g}_3 \times \underset{\sim}{g}_1 ,$$

and

$$\underset{\sim}{g} = \underset{\sim}{G}\, \underset{\sim}{x} ,$$

where

$$\underset{\sim}{G} = \begin{pmatrix} g_1^1 & g_1^2 & g_1^3 \\ g_2^1 & g_2^2 & g_2^3 \\ g_3^1 & g_3^2 & g_3^3 \end{pmatrix} = \begin{pmatrix} \underset{\sim}{g}'_1 \\ \underset{\sim}{g}'_2 \\ \underset{\sim}{g}'_3 \end{pmatrix} .$$

If α and β are the angles of $\underset{\sim}{n}$ with the $\underset{\sim}{g}_1$ and $\underset{\sim}{g}_2$ axes, respectively,

$$\cos \alpha = \underset{\sim}{n}^0 \underset{\sim}{g}_1 ,$$

$$\sin \alpha = \underset{\sim}{n}^0 \underset{\sim}{g}_2 = \cos \beta .$$

We can also say that

$$\begin{pmatrix} \cos \alpha \\ \sin \alpha \\ \epsilon \end{pmatrix} = \underset{\sim}{G} \, \underset{\sim}{n}^0 ,$$

and

$$e = \lambda \cdot \epsilon .$$

The equation of the line μ_1 on the plane σ will then be

$$(\cos \alpha_1) \, g^1 + (\sin \alpha_1) \, g^2 = e_1 .$$

We have already mentioned that all the lines will not intersect at the same point because of the different errors in the observations. We could find the most probable point of intersection by the methods employed in adjusting a multiple intersection in plane surveying. We can find the most probable point either graphically, by plotting the different lines on graph paper with a scale of, say 1 : 1000 and than choosing the point that is close to the centroid of the resultant polygon; or analytically, by a least-squares approximation that uses the equation of each line according to the method of indirect observations. In the latter case, the error ellipse on the ω plane can also be obtained.

The equation of each line should enter in the solution with a different weight depending on the accuracy with which each line is determined. This will depend, first, on the accuracy with which the observations have been made, and second, on the angle of the two directions $\underline{p}$ and $\underline{q}$. Accordingly, the weight of each observation should be the sine of the angle of the two vectors $\underline{p}$ and $\underline{q}$, which is nothing else than $|\underline{p} \times \underline{q}|$.

This weighting factor, however, holds only if we assume that the observations are perfectly simultaneous. If an error is intended for the simultaneity, an additional weighting factor should be introduced, depending on the velocity components of the satellite in the direction of the n vector. As it will be proved later, this contribution is negligible most of the time.

Once the most probable position (g_o^1, g_o^2) of B on the ω plane has been determined, the direction of the line $\vec{AB}$ can be obtained. If the direction of $\vec{AB}$ is known, the corrections δ^1, δ^2, δ^3 will be approximately

$$\begin{pmatrix} \delta^1 \\ \delta^2 \\ \delta^3 \end{pmatrix} = \underline{G}' \begin{pmatrix} g_o^1 \\ g_o^2 \\ 0 \end{pmatrix} .$$

On the other hand, the values of g_o^1 and g_o^2 can already be used directly since, with a high degree of accuracy, g_o^2 will be a correction to the height of the station (more precisely, $g_o^2 \cos \frac{\phi}{2}$, where ϕ is the geocentric angle of the two stations), and g_o^1/S a correction to the azimuth of $\vec{AB}$.

The use of quasi-simultaneous observations

Strictly simultaneous observations can be obtained by observing a flashing satellite with very simple ground equipment. In the case of tracking a passive satellite, the ground equipment is necessarily complicated in order to attain perfect synchronization for the shutter operation of the cameras involved. This can be achieved by connecting the camera-control systems with a ground wire, so that they will be activated by the same signal. The connection can also be made by a radio signal, but in this case the variation of travel time for the radio signal might be an important source of error, amounting to a few milliseconds.

The Baker-Nunn camera net can be synchronized for simultaneous observations by means of the WWV time signal. The clock at each station is synchronized with the emitted WWV signal by correcting the received signal by an average value for the travel time. The camera's cycle-driving mechanism is also synchronized by the standard frequency of the same quartz crystal that runs the clock. The cameras then are set so that the shutters will be activated at an exactly predetermined time. This method, however, is not fully automatic when used with Baker-Nunn cameras, and its success depends considerably on the observer's skill. With well-trained observers, we could expect a simultaneity within one or two milliseconds, if the WWV reception is favorable at each of the stations involved.

However, if the observations are made in sequence and at equal intervals not exceeding a fraction of a minute, strict simultaneity is not necessary, since it is possible to interpolate the apparent position of the satellite from the series of observations. In the case of the Baker-Nunn cameras, an exposure is taken every 1, 2, 4, 8, or 16 seconds, depending on the velocity and brightness of the satellite; the 4-second interval is a typical value.

Indeed, the apparent orbit of an artificial satellite is quite smooth in an interval of less than a minute. The short-period perturbations arising from the even zonal harmonics do not have any appreciable variations in this rather short interval, and it is only the very high-order tesseral harmonics that might influence the motion of the satellite in this interval. Another possible source of disturbance might be that of a variable drag, if the satellite is tumbling fast and is asymmetric. Such an effect can be avoided by using a spherical satellite that also has a high perigee; in that case the drag effect will be small.

A third-degree polynomial in time for the apparent position is, in general, sufficient to fit the observed directions for a period up to a minute or so, and in most cases a second-degree is satisfactory, since with actual observations made at precisely equal intervals the second differences are constant within the noise that should be expected from the random errors in the observations. Hence, if the synchronization is not perfect and we determine "post mortem" the divergence from simultaneity, we can interpolate from the series of observations made at station B and find the position that would correspond at the time the observation was made from station A (see fig. 3a). Instead of interpolating for the exact time of a specific observation from one station, we can interpolate for any selected time in both sets of observations, preferably the time that corresponds to the mid-point of the two times at the two stations (fig. 3b). This method has the further advantage of reducing the random errors in the observations since both interpolated values represent a mean value from a set of observed quantities, provided the orbit is smooth.

A test of the method

To test the method described above, the Smithsonian Astrophysical Observatory started in 1961 to experiment with this technique. The Baker-Nunn cameras were modified for synchronized exposures, and the observers were trained in this operation.

As a first experiment, we attempted simultaneous observations between the stations of Arequipa, Peru, and Villa Dolores, Argentina. These stations were selected for various reasons. The distance between them is one of the shortest in the Baker-Nunn system, which permits more favorable satellite passes, and the stations have a fairly reliable timing system, the reception of WWV signals being quite good. The Villa Dolores station is connected to the Argentina datum, but not to the North American datum, to which the Arequipa station and the other American stations have been tied. Thus although the distance between the two stations is small, we could expect a rather large uncertainty in the relative position of the Argentinian station, and therefore an improved determination of position.

Satellite 1961 δ1 (the 12-foot balloon) was selected as the target because of the good images we obtained for this object, and because of its spherical shape and its high apogee.

A special ephemeris had to be made for this test. The overlapping period during which the satellite was visible from both stations was first determined. Depending on the length of this period, one or more series of observations were predicted and the camera settings for each series were computed and sent to the stations. The cameras were then synchronized so that the shutters would open almost simultaneously with the emitted WWV signal.

The films were precisely reduced at the Smithsonian Astrophysical Observatory in Cambridge (see Veis, 1961) to provide observations of high accuracy. The only difference from the routine reduction on other satellites was that all frames in the overlapping period were reduced and used for the interpolation.

About 70 percent of the predictions sent to the stations did not result in observations, owing mainly to weather problems and to malfunction of minor equipment. Some 40 percent of the arcs actually photographed could not be reduced to the needed accuracy because of timing problems and, occasionally, poor image quality. These statistics, however, are based on a rather limited amount of data.

After the first successful sets of simultaneous observations were obtained from Peru and Argentina, the stations in New Mexico, Florida, and Curaçao participated in this test. Thus we had a chain of lines, whose directions can be determined in an absolute system, from New Mexico to Argentina. In addition to satellite 1961 $\delta 1$, we also used Satellite 1961 $\alpha\delta 1$, but to a lesser extent. Four pairs of simultaneous observations on satellite ANNA, the flashing geodetic satellite, have also been included in this test.

The reduction of the observations

The observed directions of Baker-Nunn camera observations, or rather their interpolated values, are expressed in right ascension and declination referred to the equator and equinox of 1950.0, while the times are given in the A-1 (atomic time) system (Veis, 1961b). These directions have to be converted to a fixed-earth, or terrestrial, system to be used according to the method described above. It is important to emphasize that the final direction obtained for the line connecting the two stations will be in the same terrestrial system as the one used to reduce the observed directions.

The series of observed positions α, δ, from each station was used to interpolate for the fictitious simultaneous observations from the two stations. The positions obtained were corrected for (a) the aberrational effect caused by the velocity of the satellite, and (b) the parallactic refraction, or the difference between the amount of refraction of the star background and the satellite. Both corrections were made in the manner described by Veis (1960). An approximate ephemeris of the satellite was used to give the range of the satellite since the accuracy needed is only of the order of 100 km. In the case of flashes of satellite ANNA there was no correction applied for aberration since the observations refer to the time of the flash.

The corrected values for α and δ give true geometric directions expressed in the celestial system. Their reduction to a terrestrial system was made according to the methods described by Veis (1963). The α's and δ's were converted to direction cosines and then premultiplied three times by the appropriate matrices for the effect of the precession, the nutation, and the rotation of the earth. This was done according to equation (19) of Veis (1963). However, the small effect of the actual motion of the pole on the earth's surface was not taken into account at this stage since the accuracy of the over-all work did not require it. The maximum effect is only 2 meters.

The sidereal angle was computed in accordance with the U.S. Naval Observatory Bulletins that give the relation between A-1 and UT-1. However, since the UT-1 given in these Bulletins refers to the meridian of Washington, it had to be reduced to the meridian of Greenwich using the nominal longitude of $\lambda = 5^h\ 08^m\ 15^s.729W$ adopted by the U.S. Naval Observatory starting 1962. All observations made in 1961 were reduced to the new longitude.

Results obtained

25 pairs of successful simultaneous observations were used between the station at Arequipa (9007) and that at Villa Dolores (9011); 13 between the station at New Mexico (9001) and that at Florida (9010); eight between the station at Curaçao (9009) and that at Florida (9010); and 10 between the station at Arequipa (9007) and that at Curaçao (9009). Of the total of 56 pairs used, 35 refer to observations of Satellite 1961 $\delta 1$ made during the period June 1961 to March 1962. Seventeen pairs refer to observations made on satellite 1961 $\alpha\delta 1$ during the period May 1962 to July 1962. Finally four pairs between stations New Mexico and Florida refer to flashes of ANNA made November 17th, 1962. The four lines corresponding to those observations are indicated by an *F* on the graph.

The graphs giving the position lines on the ω planes for the determination of the directions of the four lines connecting the above-mentioned pairs of stations appear in figures 4 to 7. The most probable position for the intersection of the lines obtained by a least-squares approximation, as well as the associated error ellipse, are shown in each graph. Three different thicknesses are used for the lines in the graph to indicate the different weights of each line.

Table 1 summarizes the results obtained and gives the following information:

1. assumed direction cosines c_o^{-1} of the line connecting the two stations referred to the geodetic system (Veis, 1961a) and based on the following coordinates.[3]

Station	X (in mm)	Y (in mm)	Z (in mm)
9001	-1.535746	-5.167221	3.401154
9007	1.942732	-5.804281	-1.796792
9009	2.251811	-5.817127	1.327282
9010	0.976289	-5.601609	2.880357

2. assumed length $|\bar{c}|$ of the line connecting the two stations based on the above coordinates;

3. the number ν of the position lines used for the determination of intersection;

4. the coordinates g_o^1 and g_o^2 of the most probable point for the intersection of the lines on the ω plane as obtained from the least-squares approximation, which is the same with the relative displacement of the second station with respect to the first on the ω plane;

5. the standard deviation of unit weight σ_o of one pair of observations on the ω plane; it is obtained from the discrepancy from a unique intersection of the position lines and corresponding to two lines intersecting at right angle.

6. the semimajor a and semiminor b axes of the error ellipse on the ω plane, as well as the angle θ between the semimajor axis and g_1 axis; these three quantities give indirectly the variance-covariance matrix of the solution;

7. the correction to the azimuth ΔA, of the line connecting the two stations and the relative correction of the elevation ΔH, of the second station with respect to the first, and their standard deviations;

8. the corrected direction cosines c_o^1 for the line connecting the two stations after the adjustment.

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[3] Some minor corrections were made later on some of the coordinates given in Special Report No. 59. The values used are given to eliminate any ambiguity.

Table 1

		9007 → 9011	9001 → 9010	9007 → 9009	9009 → 9010
	$\bar{c}_o^1$	+0.1850854	+0.9654362	+0.0984531	-0.6310564
1	$\bar{c}_o^2$	+0.4871154	-0.1669459	-0.0040919	+0.1066262
	$\bar{c}_o^3$	-0.8534998	-0.2001550	+0.9951333	+0.7683740
2	$\lvert\bar{c}\rvert$	1.826232 mm	2.601969 mm	3.139352 mm	2.021249 mm
3	ν	25	13	8	10
4	g_o^1	-130 ± 10m	+15 ± 5m	+27 ± 7m	-5 ± 9m
	g_o^2	-86 ± 13m	-10 ± 12m	-23 ± 3m	+20 ± 20m
5	σ_o	± 22m	± 14m	± 9m	± 21m
		(1.2×10^{-5})	(0.5×10^{-5})	(0.3×10^{-5})	(1.0×10^{-5})
	a	12.8m	11.8m	7.0m	20.4m
6	b	6.7m	4.3m	2.6m	8.6m
	θ	$102\overset{\circ}{.}9$	$80\overset{\circ}{.}7$	$60\overset{\circ}{.}3$	$98\overset{\circ}{.}2$
7	ΔA	$+14\overset{''}{.}7 \pm 0\overset{''}{.}8$	$-1\overset{''}{.}1 \pm 0\overset{''}{.}4$	$-1\overset{''}{.}8 \pm 0\overset{''}{.}5$	$-0\overset{''}{.}5 \pm 0\overset{''}{.}9$
	ΔH	-85 ± 13m	-10 ± 12m	-22 ± 3m	+20 ± 20m
	c_o^1	+0.1850041	+0.9654376	+0.0984426	-0.6310519
8	c_o^2	+0.4871405	-0.1669387	-0.0040879	+0.1066184
	c_o^3	-0.8535031	-0.2001544	+0.9951344	+0.7683787

Discussion of the results and general remarks

Of the determined directions for the four lines, only the one between stations 9007 and 9011 was determined from a sufficient number of observations to be of any value; the rest give only an indication of what can be obtained in the near future.

The correction obtained for the assumed direction of the line 9007-9011 indicates that the adopted value for the position of the Argentine station, which is based on zero deflection of the vertical at the origin of the Argentine Datum, was not correct, at least with respect to the North American Datum (NAD). However, this result depends not only on the adopted deflection of the vertical at the origin of the geodetic systems, but also on the parameters of the reference ellipsoid used to connect the independent geodetic systems. The adopted International Ellipse for the old System of the Smithsonian Astrophysical Observatory does not represent reality sufficiently well, and the use of a reference ellipsoid with $a = 6\ 378\ 165$ m and $f = \frac{1}{298.3}$, which is believed to be much closer to the ideal one, reduces the total correction to about one-fourth. The position of the ϖ plane of station 9011 as obtained with the above reference ellipsoid is indicated as A in figure 4.

The existence of corrections obtained for the adopted directions between stations 9001 and 9010 and 9007 and 9009, if confirmed by additional new observations (the present solution is based on an insufficient number of observations) cannot be explained by the use of an erroneous reference ellipsoid, since all stations are connected to the same North American Datum. The only explanation will be either that the NAD is not consistent as a net and the local errors introduce an erroneous orientation for the different lines, or that the orientation of the NAD as a whole is not correct. We assume here no error for the connection of the stations to NAD, and the altitude of the stations above the geoid.

According to AMS Technical Report No. 27 (AMS 1959), the uncertainties in the horizontal relative positions between stations 9001 and 9010, 9010 and 9009, and 9009 and 9007 are 9, 10, and 16 meters, respectively, and the corrections obtained so far are of the same order. Hence, even if the present results were final, they could not prove any systematic error for the orientation of the NAD. However, when final results have been obtained, they can certainly be used to improve the existing triangulation net of the NAD.

If each observed direction from each station has an uncertainty of $\pm 2''$ (10^{-5}), assuming that this is a random error and we ignore the uncertainty in time for the reduction to simultaneity, the orientation of each plane will be determined with an uncertainty of $\pm 2'' \sqrt{2} \cong \pm 3''$ (1.5×10^{-5}). In addition to other sources of errors, we should expect a final uncertainty of less than $\pm 4''$ ($\pm 2 \times 10^{-5}$), much of which will be of a random character. Combining a sufficient number of such planes (or position lines) we can expect a final uncertainty of $\pm 1''$ (0.5×10^{-5}), or better. This result, however, is based on the assumption that there are no sources of important systematic errors. The results obtained for the line 9009-9011 agree well with the above, and prove also that the Baker-Nunn observations are indeed accurate to $\pm 2''$.

Since the satellites can be observed only while above the horizon (in practice, 15° above), we should expect that the planes, and thus the position lines, will intersect within an angle not exceeding 120°, and most of the time a little more than 90°. Hence we should expect to obtain the position of the unknown station along the g^2 axis less accurately than that along the g^1. Similarly, this method will determine the relative heights with much less accuracy than it will the azimuths. This is quite obvious in the case of the line 9007-9011 where the semimajor axis of the error ellipse is oriented, practically speaking, along the g^2.

If one of the two stations is known and connected to a datum and the other is unknown, the determination of the absolute direction of the line connecting the two stations gives valuable information even though it does not solve the entire problem. Since this method can give the absolute relative heights between two points, it is possible to connect geoids where a direct connection by astrogeodetic leveling is not feasible (fig. 8). Except for the gravimetric method, this is actually the only method that can connect geoids separated by a large water gap.

When the direction of two lines connecting an unknown station with two known stations is determined, the position in space of the unknown station can be found as the intersection of the two lines. In this case the additional restriction, that the two lines should intersect, must also be taken into consideration. Or the position of the unknown station can be obtained as the mid-point of the shortest distance between the two lines, which is the usual practice in photogrammetric aerial triangulation. However, in this case there is no need to use this technique since it is possible to use directly the three-dimensional triangulation scheme described in Veis (1960).

The determination of absolute directions in space with such a technique will be very useful to orient large triangulation nets not only in azimuth, but also in elevation in terms of geoid heights. These directions will be much more effective in three-dimensional geodesy than the Laplace azimuths of classical geodesy, since they give two components instead of one, and refer to much longer lines.

The Smithsonian Astrophysical Observatory expects to continue this method of quasi-simultaneous observation and extend it to additional lines between other stations so that these directions could be used for geodetic orientation. A few lines so oriented in the European datum could provide an excellent basis for the orientation of the New European datum.

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References

KUKKAMAKI, T. J.

1959. Stellar triangulation. Bulletin Géodésique, no. 54, p. 53.

U. S. ARMY CORPS OF ENGINEERS

1959. Army Map Service Technical Report Number 27, U. S. Army World Geodetic System, 1959, part I: Methods, Project No. MO-011, November 1959.

VEIS, G.

1960. Geodetic uses of artificial satellites. Smithsonian Contributions to Astrophysics, Volume 3.

1961a. The positions of the Baker-Nunn camera stations. Smithsonian Astrophys. Obs., Special Report No. 59, 5 pp.

1961b. Catalog of precisely reduced observations No. P-I. Smithsonian Astrophys. Obs., Special Report No. 82, 28 pp.

1963. Precise aspects of terrestrial and celestial reference frames. Smithsonian Astrophys. Obs., Special Report No. 123. Also in "The Use of Artificial Satellites for Geodesy" (in press).

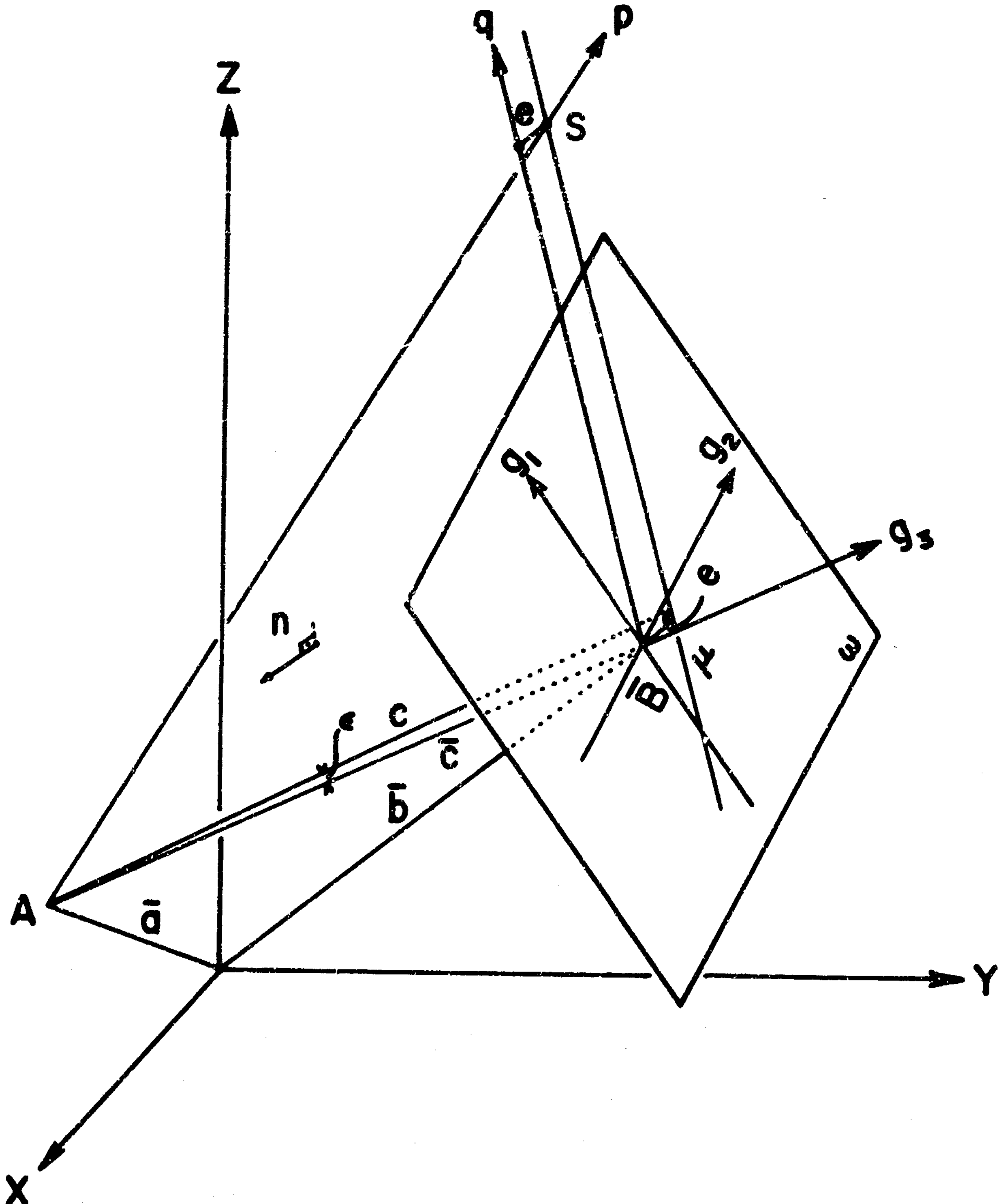


Figure 1

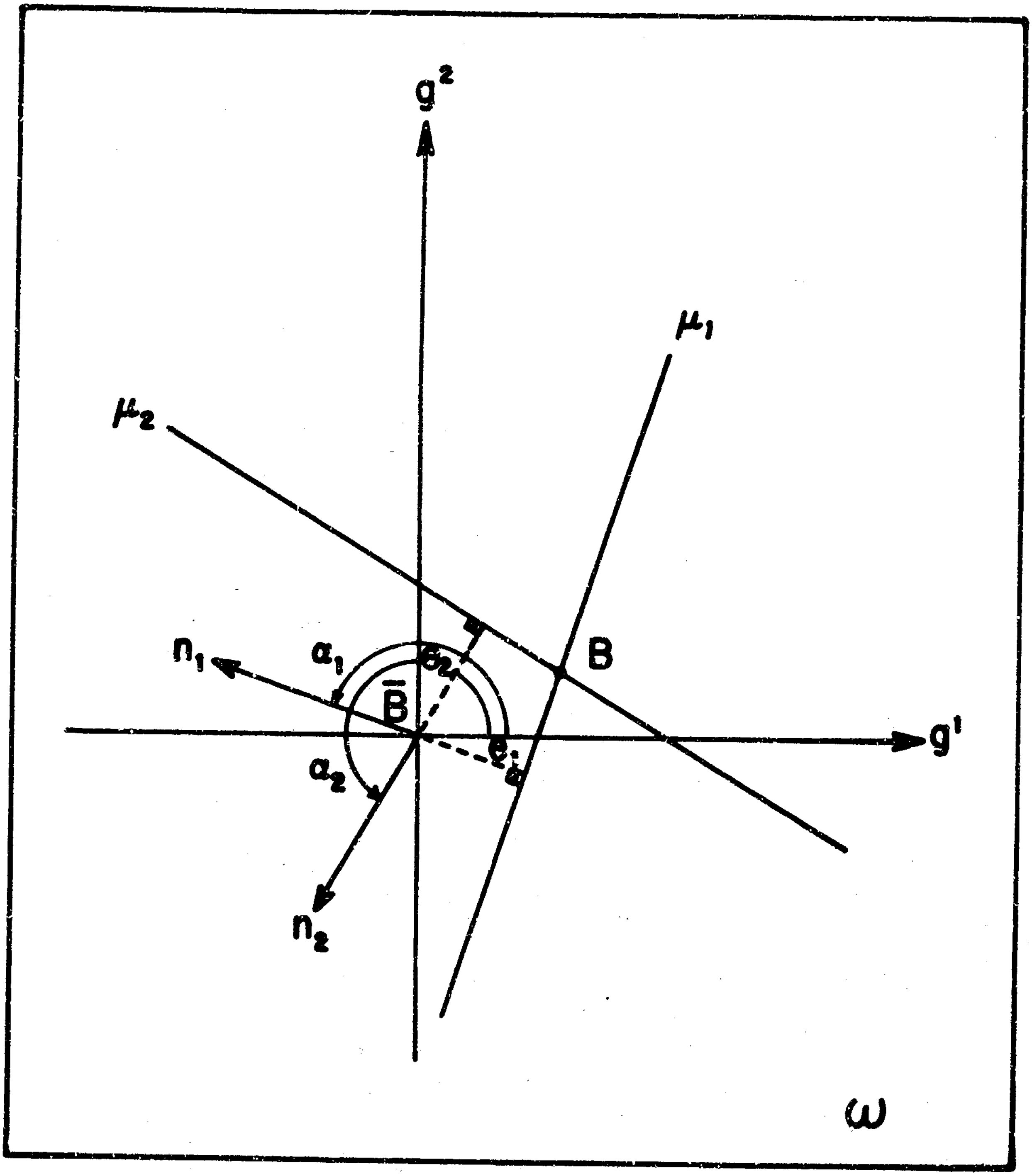


Figure 2

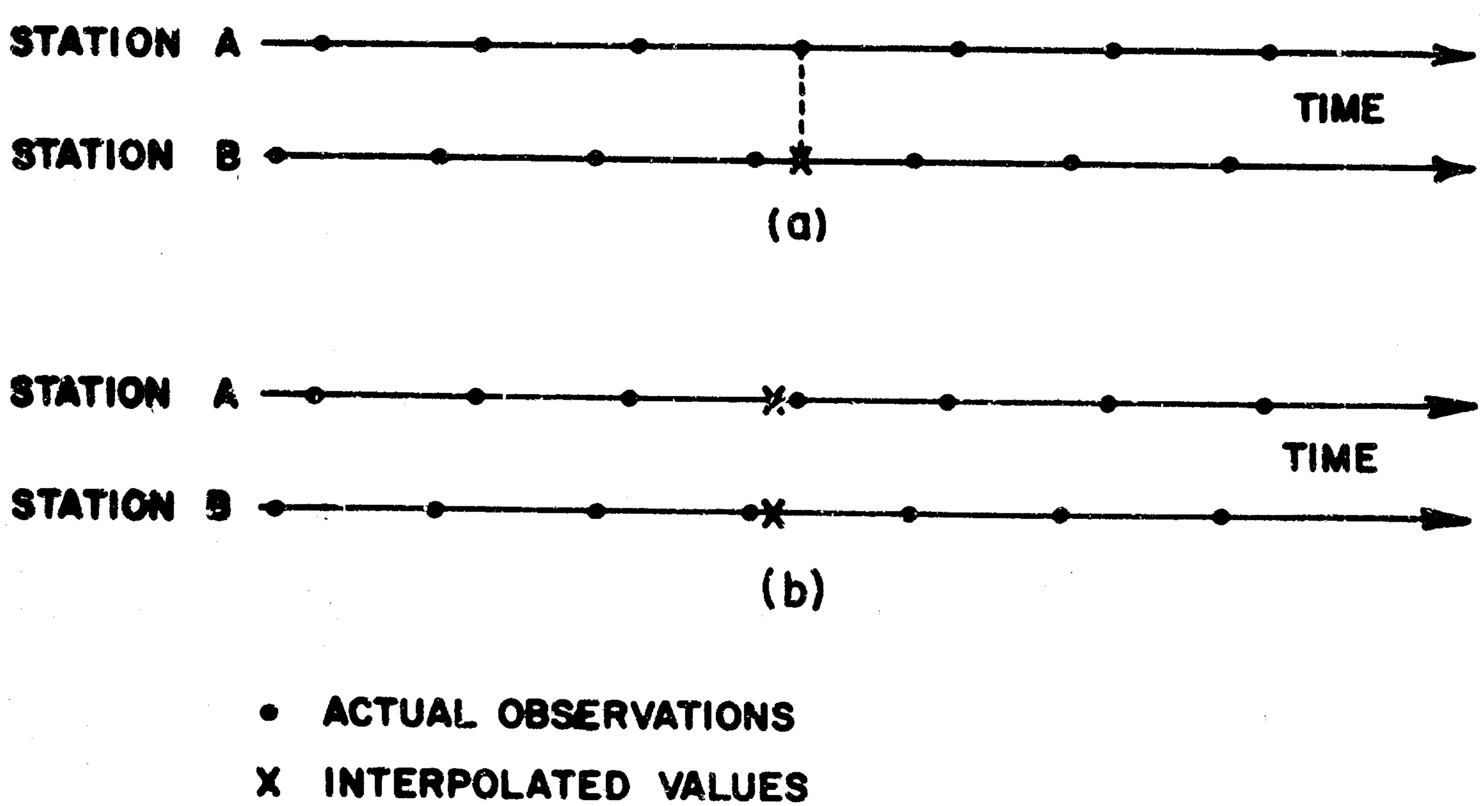
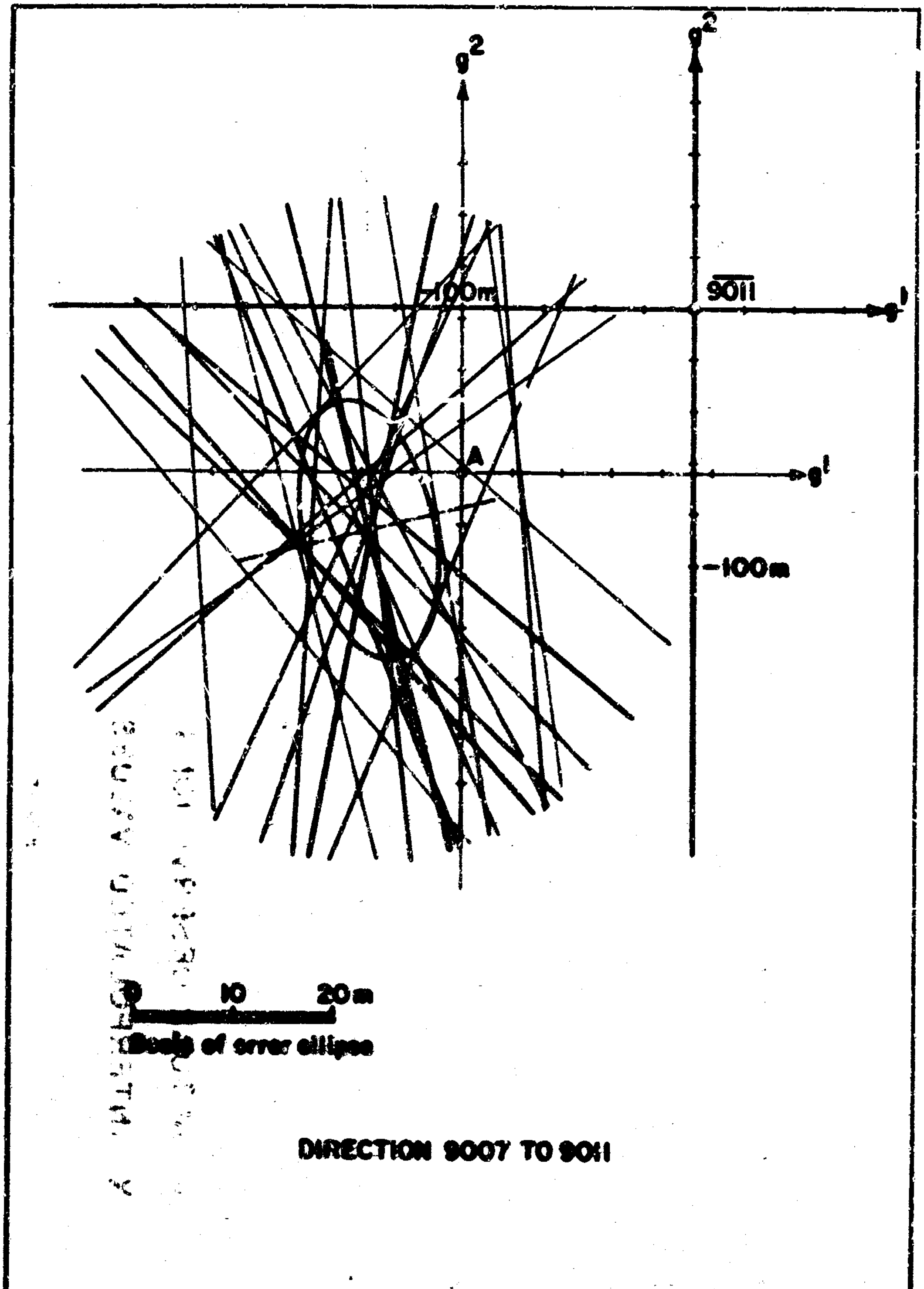
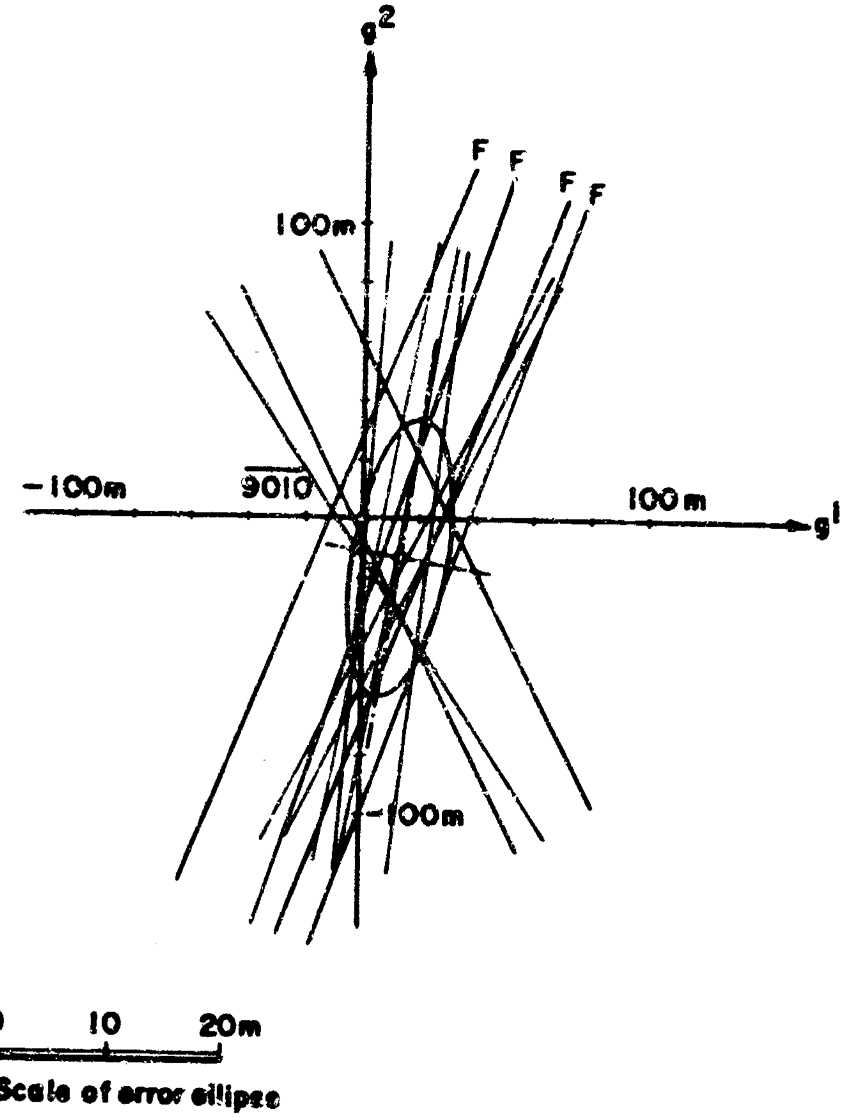

STATION A
STATION B
TIME
(a)
STATION A
STATION B
TIME
(b)
• ACTUAL OBSERVATIONS
X INTERPOLATED VALUES

Figure 3



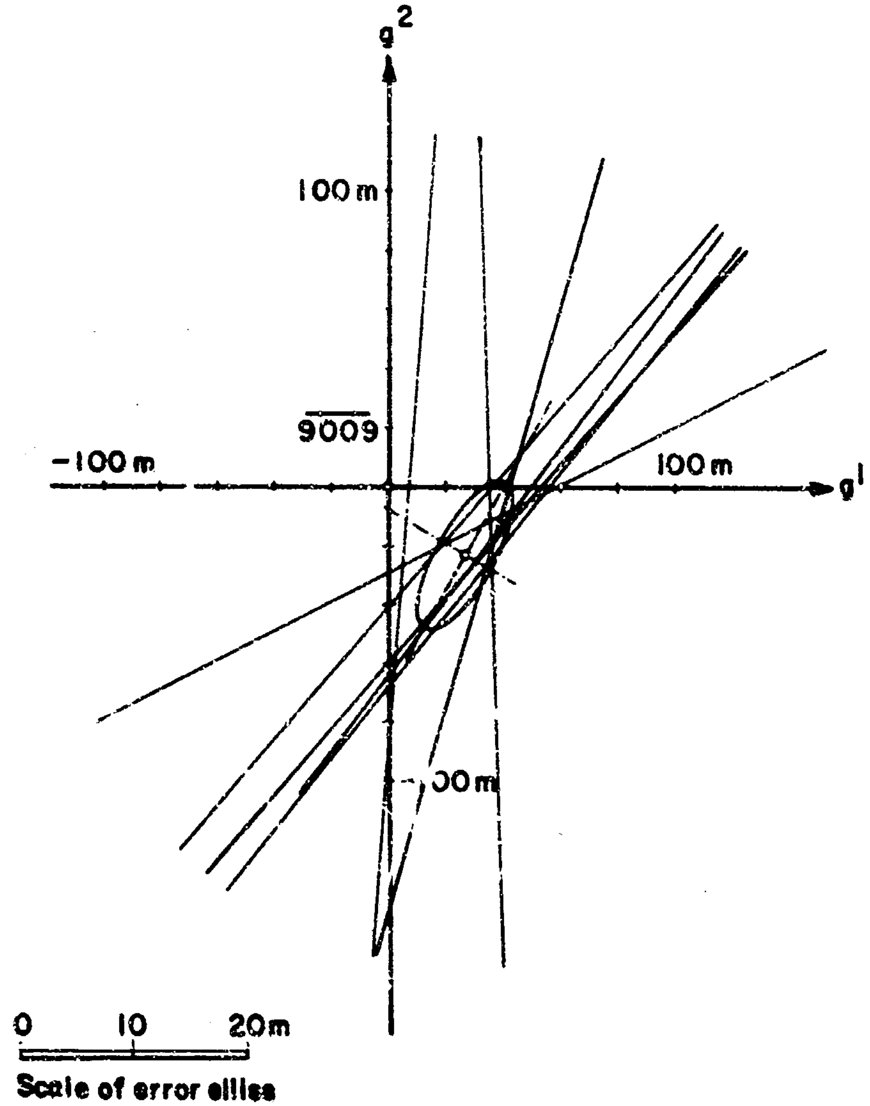
g^2
g^2
$\overline{9011}$
g^1
-100m
A
g^1
-100m
0 10 20m
Scale of error ellipse
DIRECTION 9007 TO 9011

Figure 4



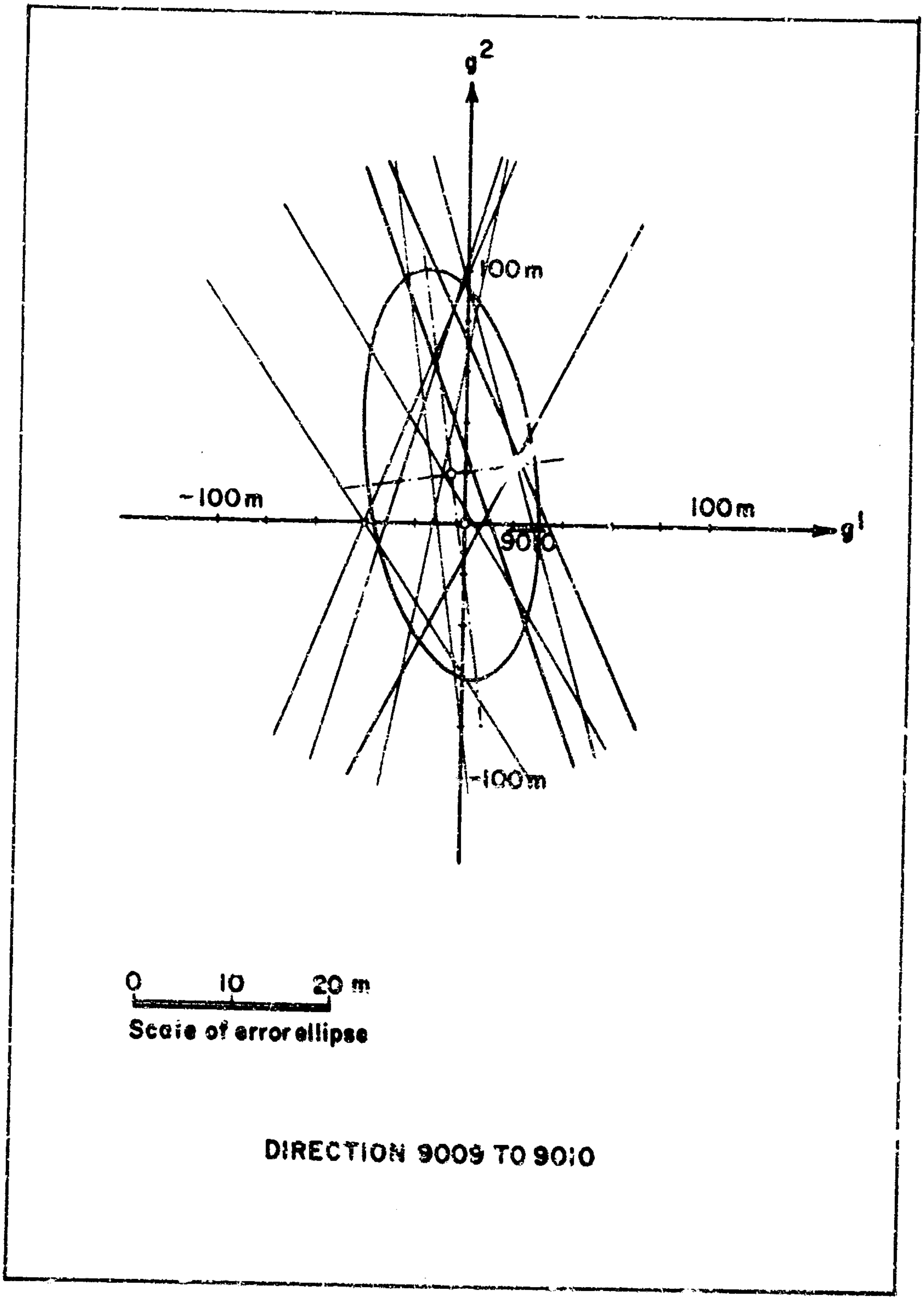
DIRECTION 9001 TO 9010

Figure 5



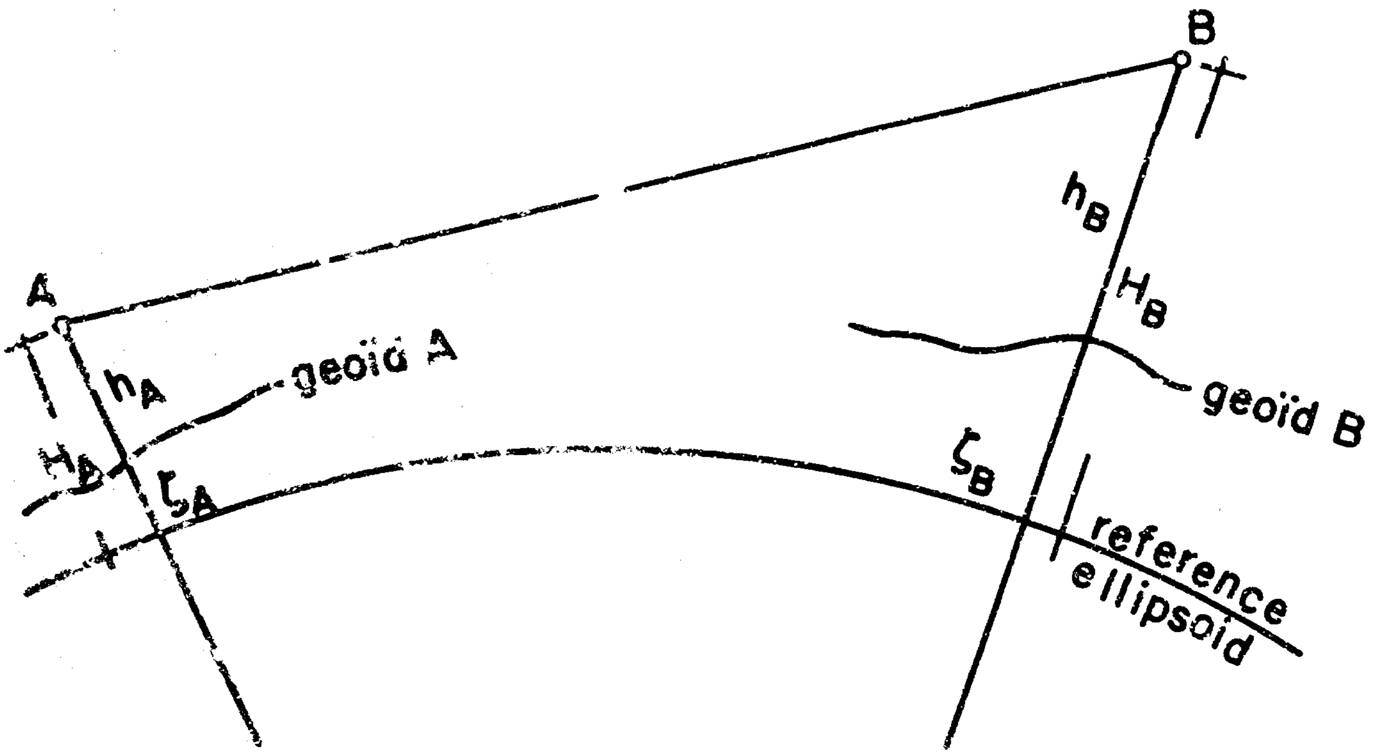
DIRECTION 9007 TO 9009

Figure 6



g^2
100 m
−100 m
100 m
g^1
9010
−100 m
0 10 20 m
Scale of error ellipse
DIRECTION 9009 TO 9010

Figure 7



B
A
h_B
H_B
geoid B
h_A
geoid A
H_A
ζ_A
ζ_B
reference
ellipsoid

Figure 6